Teaching and Learning Mathematics

Forum for Action: Effective Practices in Mathematics Education
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Current math instruction focuses on logic, critical thinking and problem solving as well as procedural knowledge, skill development and computational fluency—teaching for understanding.
My Research

• Primarily focused on the development of algebraic thinking in students from Kindergarten to Grade 9

• Documentation of the relationship between the design of lesson sequences, student activity, and an assessment of student learning during and following instruction

• Classroom-based in collaboration with educators and other researchers
What I Have Learned

• Students understand complex mathematical concepts when they are given the opportunity to construct their understanding rather than relying on rote memorization
Teaching and Learning Mathematics

• Designing Instruction (Associated Technology: CLIPS)
  – Sequenced tasks
  – Opportunities to practice procedures and review skills
  – Prioritizing visual and numeric representations
  – Emphasizing the interrelationship of representations

• Orchestrating Learning (Associated Technology: CSCL)
  – The importance of conjectures and justifications
Sequence Tasks

• Sequencing tasks means that the complexity of the mathematics is incrementally increased
• Provides scaffolding so students are supported to construct mathematical understanding by bringing together theories, experiences and previous knowledge
• Although sequenced, each task is open-ended, providing multiple points of access
**Sequence Tasks**

- Multiple opportunities to engage in similar activities allows students to practice procedural skills and to develop computational fluency.

- For example – building patterns and guessing the rules for patterns strengthens students’ multiplicative understanding as well as rapid recall of multiplication facts.
My rule is:

Output = Input x 3
My rule is:

Output = Input x 2 + 3
flowers = paving stones \times 4 + 2

flowers = paving stones \times 2 + 6

flowers = paving stones \times 2 + 2
$2x + 16 = 5x + 1$

$15 \div 3 = 5$
$\mathcal{O} = \text{graph}$

$\bullet = \text{Position}$

$\square = \text{Height}$
$x + 2$
7 of them were positive. Here is 35.

we counted 8 rules. How many rules can you think of that will intersect on position 35?
Equally Prioritizing Visual Representations

• Mathematicians have long been aware of the value of diagrams, models and other visual tools for teaching, and for developing mathematical thinking.
Equally Prioritizing Visual Representations

- Despite the obvious importance of visual images in human cognitive activities, visual representation remains a second class citizen in the teaching and learning of mathematics.
Visual Representations and Algebraic Reasoning

• Students who work with visual patterns and diagrams are more successful at understanding algebraic relationships, finding generalizations, and offering justifications than students who are taught to manipulate symbols or memorize algorithms (e.g., Beatty, 2011; Watson, 2010; Beatty & Moss, 2006; Lannin et al., 2006; Hoyles & Healy, 1999)
Equally Prioritizing Visual Representations

- Study with 31 Grade 4 students (Beatty & Moss, 2006)
- 16 students used primarily visual representations as site for problem solving
- 15 students constructed ordered tables of values and used memorized strategies
Equally Prioritizing Visual Representations

• Results of the post-test indicated the algebraic reasoning of all students improved.
• Results of a retention test given 7 months later revealed that the students who used visual representations retained more understanding.
Interactions Among Representations

• Beyond numbers, pictures and words
• Focus on how representations illustrate, deepen, and connect student understanding
  – What does the linear growing pattern representation illustrate about “steepness” for example? (relationship between tile building and graphing)
Incorporating Technology (CLIPS)

- Study of how CLIPS (computer based interactive learning objects) supports students with learning disabilities (Beatty & Bruce, 2012)
- Combines a proven visually-based curriculum with the unique properties offered by digital technology
Incorporating Technology

• CLIPS includes instructional components identified by many researchers as vital for students with LD (Fuchs et al., 2008; Fuchs et al., 2007; Montague, 2007)
  1. Focusing attention
  2. Student interaction with *dynamic* representations to construct understanding
  3. Multiple opportunities for practice
  4. Modeling with representative examples
  5. Immediate leveled corrective feedback
Incorporating Technology

The rules for Patterns A and B have different constants so the trend lines have different vertical intercepts.

Pattern A
Pattern Rule Representation
(Number of Tiles) = (Position Number) \times (2 + 4)
Pictorial Representation

Pattern B
Pattern Rule Representation
(Number of Tiles) = (Position Number) \times (2 + 1)
Pictorial Representation

Different vertical intercepts
Connecting Representations

Pattern Rule Representation
Click the up/down buttons.
(Number of Tiles) = (Position Number) × 3 + 4

Graphical Representation
Drag the coloured points on the graph. Click on the line or a black point.

Pictorial Representation
Add, remove, rearrange, customize tiles.
(Number of Tiles) = (Position Number) x \( \text{ } + \text{ } \)
Pattern Rule Representation

Click the up/down buttons.

(Number of Tiles) = (Position Number) \times 3 + 6

Pictorial Representation

Add, remove, rearrange, customize tiles.

Graphical Representation

Drag the coloured points on the graph. Click on the line or a black point.
Pattern Rule Representation

Click the up/down buttons.

(Number of Tiles) = (Position Number) \times 3 + 0

Graphical Representation

Drag the coloured points on the graph. Click on the line or a black point.
Algebraic Representation
_Click the up/down buttons._

\[ y = 3x + 4 \]

Pictorial Representation
_Add, remove, rearrange, customize tiles._

Graphical Representation
_Drag the coloured points on the graph._
_Click on the line or a black point._
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Incorporating Technology

• Two anticipated results
  – Increase in student achievement (linear relationships)
  – Students constructed conceptual understanding (not rote memorization)

• Two unanticipated results
  – Inclusive classroom community
  – Increase in student confidence
Incorporating Technology

• Sequenced dynamic representations of linear relationships had a positive effect on the levels of achievement of students identified as having a learning disability

• CLIPS allowed students to construct deep conceptual understanding of complex algebraic relationships rather than memorize procedures
Offering Conjectures

• Offering and evaluating conjectures are an essential part of fostering higher level thinking (Carpenter et al., 2003)

• Students can explore their own initial ideas to test and refine them
  – Is it always the case that this is true?
  – Can you think of a counter-example?
  – If we introduce a new idea, how does that affect the conjectures we already have?
Offering Justifications

• As important as generating conjectures is justifying or proving those conjectures
• Students provide reasoning and evidence to justify their thinking
• Students learn that
  – One counter-example makes a conjecture false
  – One definitive example does not prove a conjecture
Justifications

• Higher level justifications support higher level mathematical thinking
• Justifications are acceptable when they meet the criteria established in the mathematical community of the classroom
• This means that everyone from Kindergarten to Grade 12 can be encouraged to justify their solutions
Study of 50 Grade 9 Students

- 25 had spent 1 or 2 years (Grade 7 and 8) engaged in instruction that prioritized pattern building, offering conjectures, and providing justifications for their thinking
- 25 had received instruction that prioritized symbolic representations and memorizing algorithms
- Students were assessed on their ability to find generalized rules for functions presented in different contexts (patterns, word problems, graphs) (Beatty, 2012)
Grade 9 students were asked to find a rule for patterns like this:

Figure 1  Figure 2  Figure 3  Figure 4
Student Thinking

• Student who had participated in our instructional sequence
  – The tenth tree would have three triangles, so it’s ten times three and then you add 1, so it’s thirty-one. I know my rule is correct because you multiply the figure number by the group of three for the triangles – the figure number tells how many triangles there are – and then the trunk means you always add one more.
Student Thinking

- Student who had memorized algorithms
  - At one you have 4 and then you add 3 more. So it’s start with 4 and add 3. For the one hundredth it would be...maybe 101? I don’t know!
Results: Next, Near, Far Predictions

- Next
- Near
- Far

Symbols:
- Symbolic Memorization
- Visual Exploration
Most of the students who had memorized steps for manipulating symbols relied on drawing and counting and were unable to find a correct rule.

Students who had explored visual representations found a correct rule, and most used explicit reasoning (recognizing and articulating a functional relationship).
Results: Levels of Justification

• Students who had spent time exploring visual relationships offered sophisticated justifications for their solutions.
• These students also *revised their thinking* when their initial solution proved incorrect. This was not true for any of the students who had been taught through memorization and symbol manipulation.
Incorporating Technology (CSCL)

- Computer Supported Collaborative Learning
- Knowledge Forum
Knowledge Forum

- Knowledge Forum (Bereiter & Scardamalia) is a networked multimedia knowledge space.
- Knowledge building is supported through co-authored notes, and building on to ongoing discussions.
Knowledge Forum

• How does incorporating collaborative technology support the shift to classrooms as communities of mathematical inquiry? (Moss & Beatty, 2010, 2006)

• 68 Grade 4 students (2 different schools) participated in a teaching intervention, and were then invited to work on Knowledge Forum to collaboratively solve generalizing problems

• Only the students contributed to the KF database - teachers did not participate
Results

• Students created a culture where justifications of solutions were expected
• Over time they offered higher level justifications
• Students also revised their original ideas (up to 11 revisions per note)
Current Research

- Connecting Anishinaabe Agindaasowin and Western Mathematics
Project Sites

• Communities Involved (so far):
  ▪ Obashkaandagaang (Washagamis Bay), near Kenora
  ▪ Wauzhushk Onigum (Rat Portage), near Kenora
  ▪ Pikwàkanagàn, near Pembroke

▪ School Boards Involved (so far):
  ▪ Keewatin Patricia DSB
  ▪ Kenora Catholic DSB
  ▪ Renfrew County DSB
Theoretical Frameworks

• Ethnomathematics
  – Recognizing that school mathematics is one of many diverse mathematical practices and is no more or less important than mathematical practices that have originated in other cultures or societies (Mukhopadhyay et al., 2009)

• Culturally Responsive Education
  – Efforts to make education more meaningful by aligning instruction with the cultural paradigms and lived experience of students (Castagno & Brayboy, 2008)
Project Goals

• To investigate the connections between the mathematics embedded in traditional Anishinaabe activities, and the mathematics found in the Ontario curriculum, and to design and implement units of instruction based on these connections

• To assess the effectiveness of these units on the math content knowledge and self-efficacy of Anishinaabe students

• To develop a plan of collaborative engagement for Elders, parents, educators and students
Broad Research Questions

• Does the introduction of meaningful cultural contexts increase the achievement, confidence and engagement of Aboriginal students?

• How can we engage the wider community in the collaborative design and delivery of mathematics instruction?

• How do teachers learn from Anishinaabe pedagogical practices to achieve an equitable and inclusive mathematics classroom?
Research Activities

• Work with Elders, educators, community members, and parents to explore context, content and pedagogy
Research Activities

• Work with Aboriginal and Non-Aboriginal educators and Aboriginal artists to co-plan units of instruction
• Establish and document strategies to foster community engagement
Thank You!
References


References


